Homework on Combinations and Permutations (part 1)

Problem 1. Find the number of ways to form a four-letter sequence using the letters A, B, C, D and E if
a. repetitions of letters are permitted,
b. repetitions are not permitted,
c. the sequence contains the letter A but repetitions are not permitted,
d. the sequence contains the letter A but repetitions are permitted.

Problem 2. A variable name in the programming language FORTRAN is a sequence that has at most six characters such that the first character is a letter of the alphabet and the remaining characters, if any, are either letters or digits. Find the number of distinct variable names in this language.

Problem 3. There are three bridges connecting two towns, A and B. Between towns B and C there are four bridges. A salesperson has to travel from A to C via B. Find
a. the number of possible choices of bridges from A to C,
b. the number of choices for a round-trip travel from A to C,
c. the number of choices for a round-trip travel if no bridge is repeated.

Problem 4. Compute $P(8, 5)$, $P(9, 2)$ and $P(6, 6)$.

Problem 5. Find the value of the positive integer $n$ if
a. $P(n, 2) = 30$,
b. $P(n, 3) = 24 \cdot P(n, 2)$,
c. $10 \cdot P(n, 2) = P(3n - 1, 2) + 40$.

Problem 6. Show that $P(n, r + 1) = (n - r) \cdot P(n, r)$ and use this result to find the value of $n$ if $P(n, 9) = 15 \cdot P(n, 8)$.

Problem 7. Four SUVs, five sedans and six vans are to be parked in a row of 15 parking spots. Find the number of ways of parking these vehicles such that
a. the SUVs are parked at the beginning, then the sedans, and then the vans,
b. vehicles of the same type are parked en bloc (i.e., together in the same group).