Homework on Partial Orders

**Problem 1.** Suppose \( S = \{(x, y) \mid x \leq y\} \) is a relation defined on \( \mathbb{R} \). Show that \( S \) is a partial order on \( \mathbb{R} \).

**Problem 2.** Let \( X = \{a, b, c\} \) and \( S \) be the partial order defined on the powerset \( P(X) \) defined as \( S = \{(A, B) \mid A \text{ is a subset of } B\} \). List the elements of \( S \).

**Problem 3.** Draw the Hasse diagram of the partial order \( S \) in the last problem.

**Problem 4.** Let \( A = \{2, 3, 4, 6, 8, 12, 16, 24\} \) and \( S \) be the partial order relation on \( A \) defined by \( S = \{(a, b) \mid a \text{ divides } b\} \). Find

a. the minimal elements in \( A \),

b. the maximal elements in \( A \), and

c. the upper bounds of the set \( B = \{4, 6, 12\} \).

**Problem 5.** Explain why the relation \(<\), in which \( a \) and \( b \) are related if \( a < b \), is **not** a partial order.