Homework on Predicates and Quantifiers

Problem 1. Let $Q(x, y)$ denote the statement “$x$ is the capital of $y$.” What are the truth values of the following?

a. $Q($Denver, Colorado$)$

b. $Q($Detroit, Michigan$)$

c. $Q($Massachusetts, Boston$)$

d. $Q($New York, New York$)$

Problem 2. Let $P(x, y)$ be the statement “$x$ has taken class $y$,” where the universe of discourse for $x$ is the set of all students in your class and for $y$ is the set of all computer science courses at Gordon. Express each of the following quantifications in English.

a. $\exists x \exists y P(x, y)$

b. $\exists x \forall y P(x, y)$

c. $\forall x \exists y P(x, y)$

d. $\exists y \forall x P(x, y)$

e. $\forall y \exists x P(x, y)$

f. $\forall x \forall y P(x, y)$

Problem 3. Let $P(x)$ be the statement “$x$ can speak Spanish” and let $Q(x)$ be the statement “$x$ knows the computer language Java.” Express each of the following sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. For the universe of discourse for the quantifiers use the set of all students at Gordon.

a. There is a student at Gordon who can speak Spanish and knows Java.

b. There is a student at Gordon who can speak Spanish but who doesn’t know Java.

c. Every student at Gordon either can speak Spanish or knows Java.

d. No student at Gordon can speak Spanish or knows Java.