

## Gaussian Elimination Algorithm — No Pivoting

Given the matrix equation  $A\mathbf{x} = \mathbf{b}$  where  $A$  is an  $n \times n$  matrix, the following pseudocode describes an algorithm that will solve for the vector  $\mathbf{x}$  assuming that none of the  $a_{kk}$  values are zero when used for division.

Note: The entries  $a_{ik}$  (which are “eliminated” and become zero) are used to store and save the multipliers.

```
— Gaussian Elimination —
for  $k = 1$  to  $n - 1$  do
  for  $i = k + 1$  to  $n$  do
     $a_{ik} = a_{ik}/a_{kk}$ 
    for  $j = k + 1$  to  $n$  do
       $a_{ij} = a_{ij} - a_{ik}a_{kj}$ 
    endfor
  endfor
endfor

— Forward Elimination —
for  $k = 1$  to  $n - 1$  do
  for  $i = k + 1$  to  $n$  do
     $b_i = b_i - a_{ik}b_k$ 
  endfor
endfor

— Backward Solve —
for  $i = n$  downto  $1$  do
   $s = b_i$ 
  for  $j = i + 1$  to  $n$  do
     $s = s - a_{ij}x_j$ 
  endfor
   $x_i = s/a_{ii}$ 
endfor
```

## Gaussian Elimination Algorithm — Scaled Partial Pivoting

— *Gaussian Elimination* —

```
for  $i = 1$  to  $n$  do
     $s_i = 0$ 
    for  $j = 1$  to  $n$  do
         $s_i = \max(s_i, |a_{ij}|)$ 
    endfor
     $p_i = i$ 
endfor
for  $k = 1$  to  $n - 1$  do
     $r_{\max} = 0$ 
    for  $i = k$  to  $n$  do
         $r = |a_{p_i k} / s_{p_i}|$ 
        if  $r > r_{\max}$  then
             $r_{\max} = r$ 
             $j = i$ 
        endif
    endfor
    temp =  $p_k$ 
     $p_k = p_j$ 
     $p_j = temp$ 
    for  $i = k + 1$  to  $n$  do
         $a_{p_i k} = a_{p_i k} / a_{p_k k}$ 
        for  $j = k + 1$  to  $n$  do
             $a_{p_i j} = a_{p_i j} - a_{p_i k} a_{p_k j}$ 
        endfor
    endfor
endfor
```

*this block computes the array of  
row maximal elements*

*initialize row pointers to row numbers*

*this block finds the largest  
scaled column entry*

*row index of largest scaled entry*

*exchange row pointers*

*perform elimination on submatrix*

— *Forward Elimination* —

```
for  $k = 1$  to  $n - 1$  do
    for  $i = k + 1$  to  $n$  do
         $b_{p_i} = b_{p_i} - a_{p_i k} b_{p_k}$ 
    endfor
endfor
```

— *Backward Solve* —

```
for  $i = n$  downto  $1$  do
     $s = b_{p_i}$ 
    for  $j = i + 1$  to  $n$  do
         $s = s - a_{p_i j} x_j$ 
    endfor
     $x_i = s / a_{p_i i}$ 
endfor
```