What do you want your number theory course to be?

In Context and Interactive
Overview

Number Theory

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My answer:

In Context and Interactive

by Karl-Dieter Crisman

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My answer:

- In context
- Interactive

http://math.gordon.edu/ntic/

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- Around 1500 students, about 3% math majors
- Number Theory as 300-level elective (4/8, not req. for ed)
- Four-credit course, can assume intro to proof and calculus
- No more than 15 students
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My goals
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My goals

- See the *unity* of mathematics
- Have opportunity to *explore*
- Harness/be aware of modern computational power
Overview

These are only my goals.
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- Unity of mathematics – *In Context*
- Exploring with modern computational power – *Interactive*
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Other possible goals somewhat compatible (I do some of each):
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Other possible goals somewhat compatible (I do some of each):
- Proof practice/development
- True IBL
- Introduction to programming
- Introduction to cryptography/cryptology
A few more boring details:

- Computation provided by SageMath; much use of its \texttt{@interact} function with number-theoretic functionality
- Authoring began as worksheets, but eventually ported to PreTeXt (formerly known as Mathbook XML)
- Students required to buy print to \emph{write} in, use online for reference – both were heavily used, though more heavily by different people
In Context and Interactive

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- Choose as many topics with connections to previous coursework as possible
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What does *Interactive* mean?

- Explicitly asking students to use computation to extend their previous abilities and examine large cases for patterns
- Choose as many topics with the opportunity for exploration and/or understanding with technology as possible
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What sort of explicit connections?
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Many are ones which should be familiar and likely to show up.

- Abstract Algebra – Defining groups to make work with Euler’s theorem, primitive roots easier
- Geometry – Solutions of linear Diophantine equations as lines intersecting the integer lattice
- Analysis – Gauss estimating primes as \( \int_{2}^{n} \frac{dt}{\log(t)} \)
In Context

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- **Algebra** – Group operation of adding rational points on a conic section

- **Geometry** – Proving every prime of the form $4k + 1$ can be written as a sum of squares ‘with Minkowski’

- **Analysis** – Proving limits involving arithmetic functions like number of ways to write as a sum of squares
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Calculus has many more connections than we might recall!

- Tangent lines to curves help find new rational points
- “Newton’s method” helps find solutions modulo $p^2$ from solutions modulo $p$
- Combining integrals and limits to find estimates of the ‘long-term average’ of the number of/sum of divisors functions
There are other connections, and combinations.
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- Calculus and geometry combine in proving the ‘average value’ of $\tau$
- Combinatorics really provides most beautiful proof of Fermat’s Little Theorem
- Linear Algebra and systems of linear congruences can be expanded on quite a bit
Interactive

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“Question 19.3.1. For any given $n$, what is the constant $C_n$ such that $\sigma(n) = C_n \cdot n$? How big can this get?”
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Just as importantly, interactive versions of geometric proofs can quickly elucidate what the important visual features are where static versions are far harder to parse.
Here there be warnings against trying demos without internet.
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- Has lots of exploration and interactivity

Regardless of platform.
Concluding Unscientific Postscript

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But of course it’s also an advertisement for:

- The authoring tool, PreTeXt
- SageMath mathematics computation software
- Open licenses for your texts
Thanks!

Thanks are due to:

- You for coming!
- The organizers for accepting my abstract
- The SageMath and PreTeXt communities for a lot of support and opportunities/funding to learn more about each of them
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