SECOND MIDTERM REVIEW

1. Overall Info

You will have 50 minutes for the exam. There will once again be a combination of ‘computational’ problems and more theoretical proofs. The format will be extremely similar to the first test, though probably with a little more computation since we proved many difficult things, but not as many easy things, the last four weeks. Most of the proofs will be taken directly from the text or the homework; some of the computations will be new, but only variations on the homeworks. There will be indications of how much each problem is worth, so that you can pace yourself. This exam covers the pieces of Chapters 4-11 that we have done.

2. Chapter 4

Know the division algorithm, both the (full) statement and how to use it. Be able to say in a general way how the Well-Ordering Principle was used in its proof. Know how to use the Euclidean algorithm, and its statement; also know how to use it backwards (Exercise 4.8) to get \( d = ma + nb \) if \( d = (a, b) \). Be able to state the Fundamental Theorem of Arithmetic and use it to find GCDs, as well as explain why 1 is not a prime.

3. Chapter 6

Know all about congruences - what they are, how they work, what a residue class is, what the set of residues is. Be able to do modulo arithmetic \( \mathbb{Z}_m \). Also, be able to prove some of the very small Theorems 6.1, 6.2, and 6.3 which we used to define modulo arithmetic. Be able to define a group, and give some examples; given a non-group, be able to tell why it isn’t one. Know how to find the group \( U(m) \). Be able to find the order of an element or group I give you.

4. Chapters 7 and 8

Be able to tell what the highest power of 2 and 5 that divides a number is, as well as whether a number is divisible by 3 or 9. Be able to prove that 2 is irrational, and to give the cardinality of a set. Know which sets have the same cardinality as \( \mathbb{N} \), and which ones don’t.

5. Chapters 9 and 10

Know about symmetries, especially rotations and reflections. Know what is a polygon, a convex polygon, a regular polygon, and a non-polygon (and why). Be able to describe the number of angles in a polygon, and the size of each angle in a regular polygon.
6. Chapter 11

Be able to describe the symmetries of some very simple figures. Understand well the symmetry groups of the equilateral triangle and the square, including how to get the multiplication tables and arbitrary products. Be able to describe the subgroups of these groups, and which groups we are already familiar with that they are isomorphic to (and what an isomorphism is).

7. Useful Review Problems

Note: not all these were assigned; they are just useful. First be sure you know how to solve assigned exercises; if there is an area you are weak in, being able to solve these will help you.

Practice Problems 4.1, 4.3, and 4.4
Exercises 4.1, 4.3, 4.8, 4.10, 4.12, and 4.20
Practice Problems 6.2, 6.4, 6.6, 6.8, and 6.12
Exercises 6.1, 6.3, 6.5, 6.6, 6.8, 6.12, 6.19, 6.21, and 6.30
Practice Problems 7.1, 7.2, 8.4, 8.5
Exercises 7.1, 8.1
Practice Problems 9.1, 9.3
Exercises 9.2, 10.6, 10.17
Practice Problems 11.3, 11.6, 11.7, 11.9, 11.15
Exercises 11.1, 11.3, 11.6, 11.9