Math 112
First Midterm Exam

Instructions: This exam has a total of 100 points. You have 50 minutes. Please show every step, no matter how excruciating, and give the reason why it works - axiom or theorem or whatever. Please use bluebooks, and start a new page for each problem.

1. (15) A potpourri of short answers to begin your test.
   (a) Suppose we have used axioms to show that \((a + b) + cd = a + (dc + b)\). Which axioms did we use, in what order?
   (b) List the numbers in one’s digit arithmetic which have multiplicative inverses; then give both the multiplicative and additive inverses for them.
   (c) List the prime numbers less than 20.

2. (15) Now, True or False. No reasons needed.
   (a) For three integers \(a, b, c\), if \(a | c\) and \(b | c\), then \(a + b | c\).
   (b) For three integers \(a, b, c\), if \(a | b\) and \(b | c\), then \(a | c\).
   (c) Given a solution to the triangle game, if we perform the James rotation, we get another solution, but with a different side sum.
   (d) One’s Digit arithmetic satisfies both the Cancellation and Well-Ordering Axioms.
   (e) If \(0 < a < b\), then \(0 < a^2 < b^2\).

3. (15) Choose one of the following items to prove, from the axioms.
   (a) Take three integers \(a, b, c\). If \(a | b\) and \(a | c\), then \(a | (b + c)\).
   (b) Suppose we have an order on a set with addition. If \(a < b\) and \(c < d\), then \(a + c < b + d\).

4. (15) Choose one of the following items to prove, from axioms and (other) theorems. Be complete!
   (a) If \(a \neq 0\) then \(a^2 > 0\).
   (b) If \(a\) is in a commutative ring, then \((-1)a = -a\).

5. (30) Now some more short answer questions. Give reasons!
   (a) Give a solution to the triangle game with one side \(\{1, 6, 2\}\); then give another solution which you obtained via one of the transformations we discussed in class (and say which one).
   (b) Is the set of odd integers closed under addition? How about multiplication?
(e) Write two sets of twin primes, or the only set of triple primes.

(d) If $A = \{0, 6, 7\}$ and $B = \{0, 2, 4, 5\}$, write $A \cap B$, $A \cup B$, and a proper subset of $B$.

(e) Which axiom (or principle) did we use to prove that every composite number is divisible by a prime number?

(10) 6. Give the answers only.

(a) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - 1$.
   i. What is the domain?
   ii. What is the range?

(b) Let $A = \{2, 4, 7, \pi\}$ and $B = \{-1, -2, \ldots, -10\}$. Let $f : A \to B$ be defined by the set $\{(4, -2), (7, -8), (2, -1), (\pi, -1)\}$.
   i. Is $f$ a function?
   ii. If so, what is the image?
   iii. If so, is $f$ one-to-one?