Problem Example for this set of homework:

Let \( A = \{ \langle R \rangle \mid R \text{ is a regular expression describing a language containing at least one string } w \text{ that has 111 as a substring (i.e. } w = x111y \text{ for some } x \text{ and } y \} \}. \) Show that \( A \) is decidable. (Problem 4.15 on page 184)

The following TM decides \( A \).

“On input \( \langle R \rangle \):  
1. Construct a regular expression \( T \) that accepts every string that contains the substring 111.  
2. Test whether \( L(T) = L(R) \), using \( \text{EQ}_{\text{RE}} \) decider \( W \). (see below)  
3. If \( W \) accepts then accept; if \( W \) rejects then reject.”

\( \text{EQ}_{\text{RE}} = \{ \langle A,B \rangle \mid A \text{ and } B \text{ are REs and } L(A) = L(B) \}. \) 
\( \text{EQ}_{\text{RE}} \) is a decidable language.

PROOF:

\( W = “ \text{On input } \langle A,B \rangle \text{ where } A \text{ and } B \text{ are both regular expressions:} \) 
1. Convert \( A \) into an equivalent NFA \( A' \) by using the procedure for this conversion given in Theorem 1.54.  
2. Convert the NFA \( A' \) into an equivalent DFA \( A'' \), using the procedure for this conversion given in Theorem 1.39.  
3. Convert \( B \) into an equivalent NFA \( B' \) by using the procedure for this conversion given in Theorem 1.54.  
4. Convert the NFA \( B' \) into an equivalent DFA \( B''' \), using the procedure for this conversion given in Theorem 1.39.  
3. Test whether \( L(A'') = L(B''') \) using the \( \text{EQ}_{\text{DFA}} \) decider \( T \) from Theorem 4.5.  
4. If \( T \) accepts, accept; if \( T \) reject reject.”