Surfaces in Space

This section focuses on sketching surfaces in 3D.

Some will find this relatively easy and enjoyable, others may find it frustrating.

I believe, however, that each of you can learn to sketch these surfaces sufficiently well so that you understand the shape and will be able to solve calculus problems involving these surfaces.

Software may be helpful:
- Calculators
- Sage
- grapher (on Mac)
- commercial programs

Our focus will be on sketching by hand.

1. Take your time.
2. Use pencil and have a good eraser (you can "ink-over" if you want)
3. Try a sketch on scrap paper
4. Remember — the goal is not to produce a piece of art, but for you to understand the shape of the surface.
Surfaces in Space

Ex: Suppose we want to sketch $x^2 + y^2 = 1$ in 3D.
- This is a circle in the xy-plane
- Does not depend on z; circle in any plane of constant z.

Right Circular Cylinder

Ex: Sketch $z = \frac{x^2 + y^2}{2}$

1. Draw & label axes
2. Draw "trace" curves -
   a. $x = 0$
   b. $y = 0$
3. Draw ellipse at $z = 2$
4. Tidy up

Elliptic Paraboloid
Surface in Space

Ex Sketch 3y - 2z = 2
This is independent of x
and we can solve for z

\[ z = 3y - 2 \]

or

Ex Sketch \( z = y^2 - x^2 \)

\[ x = 0 \Rightarrow z = y^2 \]
\[ y = 0 \Rightarrow z = -x^2 \]

Hyperbolic Paraboloid
Surfaces in Space

Ex. Sketch the following traces for $x^2 + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$

a) $x = 0$
b) $z = 1$
c) $y = 2$