Discrete Probability

Definitions

The **probability of an event** is a number which expresses the long-run likelihood that the event will occur.

An **experiment** is an activity with an observable outcome.

Each repetition of an experiment is called a **trial**.

The result of each experiment is called the **outcome**.

The set of all possible outcomes is the **sample space**.

Example: The sample space S for the experiment "roll a fair die and observe the number on top" is the set

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Definitions

The **probability of an event *E***, which is a subset of a finite sample space S of equally likely outcomes, is

$$p(E) = |E| / |S|.$$

Example: What is the probability that when two dice are rolled they both show the same number?

Definitions

The **probability of an event *E***, which is a subset of a finite sample space S of equally likely outcomes, is

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Example: What is the probability that when two dice are rolled they both show the same number?

Solution:

$S = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$ so $|S| = 6 \times 6 = 36$ possible outcomes

$E = \{(x,x) \mid 1 \leq x \leq 6\}$ so $|E| = 6$

$p(E) = 6/36 = 1/6.$

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Example: What is the probability that 5 cards drawn at random from a deck of 52 cards will contain 3 cards of the same value?

1 2 3 4 5 6 7 8 9 10 11 12 13 14

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Example: What is the probability that 5 cards drawn at random from a deck of 52 cards will contain 3 cards of the same value?

Solution:

$$|S| = C(52,5) = 2{,}598{,}960$$

$$|E| = \underbrace{C(13,1)}_{\substack{\text{(\# of ways}\\ \text{to pick a}\\ \text{value)}}} \times \underbrace{C(4,3)}_{\substack{\text{(\# of ways}\\ \text{to pick 3}\\ \text{cards with}\\ \text{chosen}\\ \text{value)}}} \times \underbrace{C(48,2)}_{\substack{\text{(\# of ways to}\\ \text{pick the}\\ \text{remaining 2}\\ \text{cards)}}}$$

$$|E| = 58{,}656$$

$$p(E) = 58{,}656 / 2{,}598{,}960 = 0.0226.$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14

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Example: What is the probability that a coin tossed four times comes up heads exactly twice?

1 2 3 4 5 6 7 8 9 10 11 12 13 14

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Example: What is the probability that a coin tossed four times comes up heads exactly twice?

Solution:

$$|S| = 2^4$$

$$|E| = C(4,2) = 6$$

$$p(E) = 6/16 = 0.375.$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14

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Theorem: Let E be an event in a sample space S. The probability of an event E', the complementary event of E, is given by

$$p(E') = 1 - p(E)$$

Proof:

$$p(E') = |E'|/|S| = (|S| - |E|) / |S| = 1 - |E|/|S| = 1 - p(E).$$

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Example: A coin is tossed eight times. What is the probability that it comes up heads at least twice?

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Example: A coin is tossed eight times. What is the probability that it comes up heads at least twice?

Solution: Let E be the event "coin comes up heads at least twice". Then E' is the event "the coin comes up heads once or never."

$$|S| = 2^8 = 256$$

$$|E'| = C(8,1) + C(8,0) = 9$$

$$p(E) = 1 - p(E') = 1 - 9/256 = 0.965.$$

Note that you could do this the "hard" way: $|E| = C(8,2) + C(8,3) + \ldots + C(8,8) = 247$. $p(E) = 247/256 = 0.965$.

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Theorem: Let E and F be events in the sample space S. Then

$$p(E \cup F) = p(E) + p(F) - p(E \cap F).$$

This can be interpreted as "the probability that either E or F occur is equal to the sum of the probabilities that each event occurs minus the probability that both events occur."

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Example: What is the probability that a card selected at random from a deck of 52 cards is a spade or an ace?

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Example: What is the probability that a card selected at random from a deck of 52 cards is a spade or an ace?

Solution:

E = "card is a spade" F = "card is an ace"

$p(E) = 13/52 = 1/4$ $p(F) = 4/52 = 1/13$

$p(E \cap F) = 1/52$

So $p(E \cup F) = 1/4 + 1/13 - 1/52 = 13/52 + 4/52 - 1/52 = 16/52 = 4/13 = 0.3077$

1 2 3 4 5 6 7 8 9 10 11 12 13 14