Methods of Proof

Methods of Proof

Theorem
A statement that can be shown to be true.

Proof
A sequence of statements that form an argument.

Axioms or **Postulates**
Underlying assumptions, hypotheses, and previously proved theorems.

Rules of Inference
Allow one to draw conclusions from collections of assertions.

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Lemma
A simple theorem used in the proof of a more substantial theorem.

Corollary
A result that can be established directly once a theorem is proved.

Fallacies
Common forms of incorrect reasoning.

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Rules of Inference		
Rule of Inference	**Tautology**	**Name**
p therefore $(p \vee q)$	$p \Rightarrow (p \vee q)$	Addition
$(p \wedge q)$ therefore p	$(p \wedge q) \Rightarrow p$	Simplification
p and $p \Rightarrow q$ therefore q	$[p \wedge (p \Rightarrow q)] \Rightarrow q$	Modus ponens
$\neg q$ and $p \Rightarrow q$ therefore $\neg p$	$[\neg q \wedge (p \Rightarrow q)] \Rightarrow \neg p$	Modus tollens
$p \Rightarrow q$ and $q \Rightarrow r$ therefore $p \Rightarrow r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$	Hypothetical syllogism
$p \vee q$ and $\neg p$ therefore q	$[(p \vee q) \wedge \neg p] \Rightarrow q$	Disjunctive syllogism

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Examples of rules of inference

Addition:	It is sunny. Therefore it is either sunny or it is raining.
Simplification:	It is sunny and it is hot. Therefore it is sunny.
Modus ponens:	It is sunny. If it is sunny then it is hot. Therefore it is hot.
Modus tollens:	It is not hot. If it is sunny then it is hot. Therefore it is not sunny.
Hypothetical syllogism:	If it is sunny then it is hot. If it is hot then we sweat. Therefore if it is sunny then we sweat.
Disjunctive Syllogism:	It is sunny or it is raining. It is not sunny. Therefore it is raining.

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Fallacies

Affirming the conclusion:	If it is sunny then it is hot. It is hot. Therefore it is sunny.
Denying the hypothesis:	If it is sunny then it is hot. It is not sunny. Therefore it is not hot.
Circular reasoning:	(Use of statement to be proved in the proof itself.)

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Types of Proof

Vacuous proof

$p \Rightarrow q$ when p is false.

Trivial proof

$p \Rightarrow q$ when q is true.

Direct proof

$p \Rightarrow q$: asserting that p is true requires q to be true.

Indirect proof

$p \Rightarrow q$: asserting $\neg q$ is true requires that $\neg p$ is true.

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Proof by contradiction

$p \Rightarrow q$: assume that $p \Rightarrow \neg q$ is true and show that a contradiction arises.

Proof by cases

$(p_1 \vee p_2 \vee \ldots \vee p_n) \Rightarrow q$: show that each $p_i \Rightarrow q$

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