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Predicates and Quantifiers

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Predicates and Quantifiers

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To be a proposition a statement must be either **true** or **false**. Thus statements like $x>0$ are not propositions. This is because their truth value cannot be determined unless additional information is specified; in this case the value of x.

Statements like $x>0$ and $3+x=5$ have two parts: a **variable** (or **variables**) and a **predicate**.

We denote these statements $P(x)$ where $P(\ )$ is the predicate and x *is the variable.*

Once the variable (or variables) is assigned a value then the truth value of the statement can be determined.

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Let $P(x,y)$ denote the statement "$x<2+y$." What is the truth value of $P(2,1)$?

$$x < 2+y$$
$$2 < 2+1$$
$$2 < 3$$
TRUE

$P(x)$ and $P(x,y)$ are called **propositional functions**.

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When does a propositional function become a proposition?

1. when all variables are assigned values
2. when the truth of the function can be determined for all values of the variable in the **universe of discourse**.

The second form above is called **quantification**. Two types of quantification are important:

1. The **universal quantification** of $P(x)$ is the proposition $\forall x\, P(x)$, which is read "for all x, P of x (is true)."
2. The **existential quantification** of $P(x)$ is the proposition $\exists x\, P(x)$, which is read "there exists an x such that P of x (is true)."

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1. If $P(x)$ is $x^2>0$ and the universe of discourse is the real numbers, determine the truth values of both the universal and existential quantifications.

2. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the universe of discourse for x is the set of students. Express each of the following quantifications in English:

   a. $\exists x\, P(x)$ (answer)
   b. $\forall x\, P(x)$ (answer)
   c. $\exists x\, P'(x)$ (answer)
   d. $\forall x\, \neg P(x)$ (answer) (another answer)

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Let $T(x,y)$ be the propositional function "x is at least as tall as y." The universe of discourse consists of three students:

Garth	5’ 11’’
Erin	5’ 6’’
Marty	6’ 0’’

Express in words the following propositions and determine their truth value:

a. $\forall x\, \forall y\, T(x,y)$ (answer)
b. $\forall x\, \exists y\, T(x,y)$ (answer)
c. $\exists x\, \forall y\, T(x,y)$ (answer)
d. $\exists x\, \exists y\, T(x,y)$ (answer)

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Let $L(x,y)$ be the proposition "x loves y" and let the universe of discourse be the people in the world. Express the following statements as quantifications.

a. Everybody loves Jerry (answer)
b. Everybody loves somebody (answer)
c. There is somebody that everybody loves (answer)
d. Nobody loves everybody (answer)
e. There is somebody that Lydia does not love (answer)
f. There is somebody that no one loves (answer)
g. There is exactly one person that everybody loves (answer)
h. There are two people that Lynn loves (answer)
i. Everyone loves himself or herself (answer)
j. There is someone loves no one besides himself or herself (answer)

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