Sets

MAT231

Transition to Higher Mathematics

Fall 2014

Outline

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    - Introduction
    - Cartesian Products
    - Subsets
    - Power Sets
    - Union, Intersection, Difference
    - Set Complement
    - Venn Diagrams
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Definition of set

Definition

A **set** is a collection of items, called **elements** or **members**. The elements of a set may be

- *abstract:* numbers, ideas, concepts
- *physical:* numerals, objects

We will usually use uppercase letters to name sets, and lower case letters as elements. The symbol $\in$ denotes membership: $x \in A$ indicates that x is an element of the set A.

Two sets A and B are **equal** if and only if they have the same elements. That is, $A = B$ if and only if every element in A is contained in B *and* every element in B is contained in A.

Specifying sets: Enumeration

There are many ways to specify a set. The simplest way is to list the elements contained in the set, as in $1, 2, 3$. This is called **enumeration**. We use braces $\{$ and $\}$ to bracket the list of elements in a set.

Example

- $\{1, 2, 3\}$
- $\{t, u, v, w, x, y, z\}$
- $\{☺, ☹\}$

For sets, membership is all that matters and the order the elements are listed in is insignificant. Thus $\{a, b, c\}$ is the same set as $\{c, b, a\}$.

Specifying sets: Enumeration

Listing sets is inconvenient if there are a large number of elements in them. If there is an easy-to-see pattern then an *ellipsis* can be used.

Example

- $\{1, 2, 3, \ldots, 10\}$
- $\{0, 2, 4, 6, \ldots\}$
- $\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$

Specifying sets: Set-builder notation

Sets can also be specified using **set-builder notation**. It provides a *expression* for a typical element in the set and then one or more *membership rules*.

Example

- $\{x : x \text{ is an even integer}\}$
- $\{b : b \text{ is a butterfly}\}$

Some special sets

There are some important sets of numbers, which we now name.

- $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ is the set of **natural numbers**.
- $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of **integers**.
- $\mathbb{Q} = \{x/y : x, y \in \mathbb{Z}, y \neq 0\}$ is the set of **rational numbers**
- $\mathbb{R}$ is the set of **real numbers**.

It is not possible to specify $\mathbb{R}$ using enumeration and using enumeration for $\mathbb{Q}$ is extremely cumbersome. Set-builder notation, however, makes specifying $\mathbb{Q}$ relatively easy.

Another important set is the **empty set**, denoted $\{\}$ or $\varnothing$. This is the set that contains no elements.

More examples of set-builder notation

Here are some more examples using set-builder notation:

Example

- $\{3n : n \in \mathbb{Z}\} = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$
- $\{2^{n-1} : n \in \mathbb{N}\} = \{1, 2, 4, 8, 16, \ldots\}$
- $\{x \in \mathbb{Z} : |x - 2| < 5\} = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

Cardinality

The **cardinality** of a set is the number of elements in the set. The cardinality of a set X is denoted $|X|$.

Example

- $|\{a, b, c\}| = 3$
- $|\{x : x \text{ is a letter of the alphabet}\}| = 26$
- $|\varnothing| = 0$ (this is the only set with cardinality zero)
- $|\{x \in \mathbb{Z} : x^2 \leq 12\}| = 7$

A **finite** set has a finite number of elements, while a set having infinitely many elements is called an **infinite** set.

The cardinality of a finite set is a number; the cardinality of infinite sets is usually expressed by saying what well known infinite sets have the same cardinality.

Cardinality

What are the cardinalities of the following sets:

- $A = |\{a, \{b, c, d\}\}|$
- $B = |\{\{a\}, \{b, c\}, d\}|$
- $C = |\{\{a, b, c, d\}\}|$
- $D = |\{\{a, b, \{c, d\}\}, \varnothing, \{\varnothing\}\}|$

Cardinality

What are the cardinalities of the following sets:

- $A = |\{a, \{b, c, d\}\}|$ $\quad |A| = 2$
- $B = |\{\{a\}, \{b, c\}, d\}|$
- $C = |\{\{a, b, c, d\}\}|$
- $D = |\{\{a, b, \{c, d\}\}, \varnothing, \{\varnothing\}\}|$

Cardinality

What are the cardinalities of the following sets:

- $A = |\{a, \{b, c, d\}\}|$ $\quad$ $|A| = 2$
- $B = |\{\{a\}, \{b, c\}, d\}|$ $\quad$ $|B| = 3$
- $C = |\{\{a, b, c, d\}\}|$
- $D = |\{\{a, b, \{c, d\}\}, \varnothing, \{\varnothing\}\}|$

Cardinality

What are the cardinalities of the following sets:

- $A = |\{a, \{b, c, d\}\}|$ $|A| = 2$
- $B = |\{\{a\}, \{b, c\}, d\}|$ $|B| = 3$
- $C = |\{\{a, b, c, d\}\}|$ $|C| = 1$
- $D = |\{\{a, b, \{c, d\}\}, \varnothing, \{\varnothing\}\}|$

Cardinality

What are the cardinalities of the following sets:

- $A = |\{a, \{b, c, d\}\}|$ $\quad |A| = 2$
- $B = |\{\{a\}, \{b, c\}, d\}|$ $\quad |B| = 3$
- $C = |\{\{a, b, c, d\}\}|$ $\quad |C| = 1$
- $D = |\{\{a, b, \{c, d\}\}, \varnothing, \{\varnothing\}\}|$ $\quad |D| = 3$

Cartesian products

Definition

The **Cartesian product** of two sets A and B is another set, denoted $A \times B$ whose elements are **ordered pairs**, defined by

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Example

If $A = \{a, b, c\}$ and $B = \{0, 1\}$ then

$$A \times B = \{(a, 0), (a, 1), (b, 0), (b, 1), (c, 0), (c, 1)\}$$

while

$$B \times A = \{(0, a), (0, b), (0, c), (1, a), (1, b), (1, c)\}$$

Recall that the order of elements in a set is unimportant, but the order of elements in a ordered pair is significant.

Cartesian products

If we extend the idea of an ordered pair to an **ordered triple** or even an **ordered *n*-tuple** then we can define the Cartesian product of three or more sets:

$$A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$$
$$W \times X \times Y \times Z = \{(w, x, y, z) : w \in W, x \in X, y \in Y, z \in Z\}$$

Notice that the following two sets are different. One is the Cartesian product of three sets while the other is the Cartesian product of two sets. One contains ordered triples while the other contains ordered pairs.

$$A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$$
$$A \times (B \times C) = \{(a, (b, c)) : a \in A, b \in B, c \in C\}$$

Cartesian products as powers of sets

In some applications Cartesian products of a set with itself are common. We use the notation $A^2 = A \times A$ to denote this. In general

$$\begin{aligned} A^n &= A \times A \times \cdots \times A \\ &= \{(x_1, x_2, \ldots, x_n) : x_1, x_2, \ldots, x_n \in A\} \end{aligned}$$

Example

$$\begin{aligned} \{a, b\}^3 = \{&(a, a, a), (a, a, b), (a, b, a), (a, b, b), \\ &(b, a, a), (b, a, b), (b, b, a), (b, b, b)\} \end{aligned}$$

Cardinality of Cartesian products

Question: What is $|A \times B|$?

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Answer: $|A \times B| = |A| \cdot |B|$

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Answer: $|A \times B| = |A| \cdot |B|$

Justification: Every element from A must be paired with every element from B.

Subsets

Definition

Suppose A and B are sets. If every element of A is also an element of B then we say A is a **subset** of B and denote this as $A \subseteq B$. If A is not a subset of B we write $A \not\subseteq B$. This means there is at least one element of A that is not contained in B.

Example

Which of the following sets are subsets of $A = \{1, 2, 3, 4, 5, 6\}$?

- $\{2, 4, 6\}$
- $\{0, 1, 2, 3\}$
- $\{1, 2, 3, 4, 5, 6\}$
- $\varnothing$

Subsets

Definition

Suppose A and B are sets. If every element of A is also an element of B then we say A is a **subset** of B and denote this as $A \subseteq B$. If A is not a subset of B we write $A \nsubseteq B$. This means there is at least one element of A that is not contained in B.

Example

Which of the following sets are subsets of $A = \{1, 2, 3, 4, 5, 6\}$?

- $\{2, 4, 6\}$ (**is** a subset)
- $\{0, 1, 2, 3\}$
- $\{1, 2, 3, 4, 5, 6\}$
- $\varnothing$

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Example

Which of the following sets are subsets of $A = \{1, 2, 3, 4, 5, 6\}$?

- $\{2, 4, 6\}$ (**is** a subset)
- $\{0, 1, 2, 3\}$ (**is not** a subset, 0 not contained in A)
- $\{1, 2, 3, 4, 5, 6\}$
- $\varnothing$

Subsets

Definition

Suppose A and B are sets. If every element of A is also an element of B then we say A is a **subset** of B and denote this as $A \subseteq B$. If A is not a subset of B we write $A \not\subseteq B$. This means there is at least one element of A that is not contained in B.

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- $\{2, 4, 6\}$ (**is** a subset)
- $\{0, 1, 2, 3\}$ (**is not** a subset, 0 not contained in A)
- $\{1, 2, 3, 4, 5, 6\}$ (**is** a subset, equal to A)
- $\varnothing$

Subsets

Definition

Suppose A and B are sets. If every element of A is also an element of B then we say A is a **subset** of B and denote this as $A \subseteq B$. If A is not a subset of B we write $A \not\subseteq B$. This means there is at least one element of A that is not contained in B.

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Which of the following sets are subsets of $A = \{1, 2, 3, 4, 5, 6\}$?

- $\{2, 4, 6\}$ (**is** a subset)
- $\{0, 1, 2, 3\}$ (**is not** a subset, 0 not contained in A)
- $\{1, 2, 3, 4, 5, 6\}$ (**is** a subset, equal to A)
- $\varnothing$ (**is** a subset - does not contain any element *not* in A)

The Empty set as a subset

The last example leads to an important fact: the empty set $\varnothing$ is a subset of every set. Thus $\varnothing \subseteq X$ for all sets X.

We can rephrase set equality in terms of subsets. Two sets A and B are **equal** if and only if $A \subseteq B$ *and* $B \subseteq A$.

Listing subsets

Suppose $L = \{w, x, y, z\}$. List the subsets of L.

- Clearly $\varnothing$ is a subset.

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Listing subsets

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- Clearly $\varnothing$ is a subset.
- $\{w\}$, $\{x\}$, $\{y\}$, and $\{z\}$ are also subsets of L.
- So are $\{w, x\}$, $\{w, y\}$, $\{w, z\}$, $\{x, y\}$, $\{x, z\}$, and $\{y, z\}$. These subsets all have cardinality 2.

Listing subsets

Suppose $L = \{w, x, y, z\}$. List the subsets of L.

- Clearly $\varnothing$ is a subset.
- $\{w\}$, $\{x\}$, $\{y\}$, and $\{z\}$ are also subsets of L.
- So are $\{w, x\}$, $\{w, y\}$, $\{w, z\}$, $\{x, y\}$, $\{x, z\}$, and $\{y, z\}$. These subsets all have cardinality 2.
- $\{w, x, y\}$, $\{w, x, z\}$, $\{w, y, z\}$, and $\{x, y, z\}$ are the only subsets with cardinality 3.

Listing subsets

Suppose $L = \{w, x, y, z\}$. List the subsets of L.

- Clearly $\varnothing$ is a subset.
- $\{w\}$, $\{x\}$, $\{y\}$, and $\{z\}$ are also subsets of L.
- So are $\{w, x\}$, $\{w, y\}$, $\{w, z\}$, $\{x, y\}$, $\{x, z\}$, and $\{y, z\}$. These subsets all have cardinality 2.
- $\{w, x, y\}$, $\{w, x, z\}$, $\{w, y, z\}$, and $\{x, y, z\}$ are the only subsets with cardinality 3.
- Finally, the only subset of cardinality 4 is the set L itself: $\{w, x, y, z\}$.

Listing subsets

While tedious, listing subsets is straightforward:

- Start with the empty set $\varnothing$.
- List all the **singleton** subsets (sets with one element).
- List all possible subsets with two elements.
- List all possible subsets with three elements.
- Continue until original set itself is listed as a subset.

Power Sets

Definition

If A is a set, the **power set** of A is another set, denoted $\mathcal{P}(A)$ and is the set of all subsets of A.

$$\mathcal{P}(A) = \{X : X \subseteq A\}$$

Example

- $\mathcal{P}(\{a, b\}) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$
- $\mathcal{P}(\{a, b, c\}) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

Cardinality of power sets

Suppose A is a finite set. What is the cardinality of $\mathcal{P}(A)$?

Notice that

- $|\mathcal{P}(\varnothing)| = 1$ (since $\mathcal{P}(\varnothing) = \{\varnothing\}$)
- $|\mathcal{P}(\{a\})| = 2$
- $|\mathcal{P}(\{a, b\})| = 4$
- $|\mathcal{P}(\{a, b, c\})| = 8$

Can you think of a general rule?

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- $|\mathcal{P}(\{a, b\})| = 4$
- $|\mathcal{P}(\{a, b, c\})| = 8$

Can you think of a general rule?

Fact

If A is a finite set, $|\mathcal{P}(A)| = 2^{|A|}$.

Union, Intersection, Difference

Definition

Suppose A and B are two sets.

- The **union** of A and B is the set

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

- The **intersection** of A and B is the set

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

- The **difference** of A and B is the set

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

(Sometimes $A \backslash B$ is used to denote set difference)

Operations involving two sets

Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B =$
- $A \cup C =$
- $B \cup C =$
- $A \cap B =$
- $A \cap C =$
- $B \cap C =$
- $A - B =$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
- $C - B =$

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- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C =$
- $B \cup C =$
- $A \cap B =$
- $A \cap C =$
- $B \cap C =$
- $A - B =$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
- $C - B =$

Operations involving two sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C =$
- $A \cap B =$
- $A \cap C =$
- $B \cap C =$
- $A - B =$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
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- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B =$
- $A \cap C =$
- $B \cap C =$
- $A - B =$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
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Operations involving two sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C =$
- $B \cap C =$
- $A - B =$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
- $C - B =$

Operations involving two sets

Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C =$
- $A - B =$
- $B - A =$
- $A - C =$
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Operations involving two sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C = \{2, 4\}$
- $A - B =$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
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Operations involving two sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C = \{2, 4\}$
- $A - B = \{5, 6, 7, 8\}$
- $B - A =$
- $A - C =$
- $C - A =$
- $B - C =$
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Operations involving two sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C = \{2, 4\}$
- $A - B = \{5, 6, 7, 8\}$
- $B - A = \{0, 1, 2, 3\}$
- $A - C =$
- $C - A =$
- $B - C =$
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Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C = \{2, 4\}$
- $A - B = \{5, 6, 7, 8\}$
- $B - A = \{0, 1, 2, 3\}$
- $A - C = \{5, 7, 8\}$
- $C - A =$
- $B - C =$
- $C - B =$

Operations involving two sets

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Let $A = \{4,5,6,7,8\}$, $B = \{0,1,2,3,4\}$, and $C = \{2,4,6\}$. Then

- $A \cup B = \{0,1,2,3,4,5,6,7,8\}$
- $A \cup C = \{2,4,5,6,7,8\}$
- $B \cup C = \{0,1,2,3,4,6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4,6\}$
- $B \cap C = \{2,4\}$
- $A - B = \{5,6,7,8\}$
- $B - A = \{0,1,2,3\}$
- $A - C = \{5,7,8\}$
- $C - A = \{2\}$
- $B - C =$
- $C - B =$

Operations involving two sets

Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C = \{2, 4\}$
- $A - B = \{5, 6, 7, 8\}$
- $B - A = \{0, 1, 2, 3\}$
- $A - C = \{5, 7, 8\}$
- $C - A = \{2\}$
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Operations involving two sets

Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

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- $A \cup C = \{2, 4, 5, 6, 7, 8\}$
- $B \cup C = \{0, 1, 2, 3, 4, 6\}$
- $A \cap B = \{4\}$
- $A \cap C = \{4, 6\}$
- $B \cap C = \{2, 4\}$
- $A - B = \{5, 6, 7, 8\}$
- $B - A = \{0, 1, 2, 3\}$
- $A - C = \{5, 7, 8\}$
- $C - A = \{2\}$
- $B - C = \{0, 1, 3\}$
- $C - B = \{6\}$

Operations involving three or more sets

Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $(A \cup B) \cap (A \cup C) =$
- $(A \cap B) \cup (A \cap C) =$
- $(A - B) \cup (B - A) =$
- $(A - B) \cap (B - A) =$

Operations involving three or more sets

Example

Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $(A \cup B) \cap (A \cup C) = \{2, 4, 5, 6, 7, 8\}$
- $(A \cap B) \cup (A \cap C) =$
- $(A - B) \cup (B - A) =$
- $(A - B) \cap (B - A) =$

Operations involving three or more sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $(A \cup B) \cap (A \cup C) = \{2, 4, 5, 6, 7, 8\}$
- $(A \cap B) \cup (A \cap C) = \{4, 6\}$
- $(A - B) \cup (B - A) =$
- $(A - B) \cap (B - A) =$

Operations involving three or more sets

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

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- $(A \cap B) \cup (A \cap C) = \{4, 6\}$
- $(A - B) \cup (B - A) = \{0, 1, 2, 3, 5, 6, 7, 8\}$
- $(A - B) \cap (B - A) =$

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Let $A = \{4, 5, 6, 7, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{2, 4, 6\}$. Then

- $(A \cup B) \cap (A \cup C) = \{2, 4, 5, 6, 7, 8\}$
- $(A \cap B) \cup (A \cap C) = \{4, 6\}$
- $(A - B) \cup (B - A) = \{0, 1, 2, 3, 5, 6, 7, 8\}$
- $(A - B) \cap (B - A) = \varnothing$

Disjoint sets

Two sets are **disjoint** if they have no elements in common.

Another way to say this: Sets A and B are disjoint if $A \cap B = \varnothing$.

The Universal Set

We can always consider a set to be a subset of another set. In any given situation involving sets, we assume there is a **universal set** that contains each of our sets as a subset.

Example

Consider the sets $A = \{1, 2, 3\}$, $B = \{\frac{1}{2}, \frac{3}{5}, 5\}$, and $C = \{\pi, e\}$.

- When dealing with only set A, we can use $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, or $\mathbb{R}$ as the universal set.
- When dealing with both A and B, we could consider $\mathbb{Q}$ or $\mathbb{R}$ to be the universal set.
- When dealing with all three sets we would take $\mathbb{R}$ to be the universal set.

Often the context will suggest what the universal set should be.

Set Complement

Definition

Let A be a set with universal set U. The **complement** of A, denoted $\overline{A}$, is the set

$$\overline{A} = U - A.$$

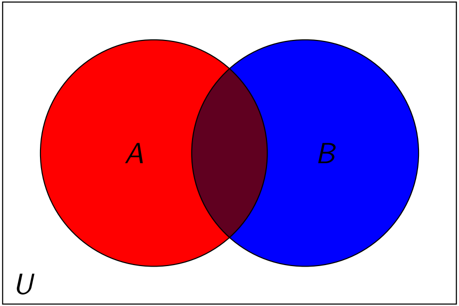
Essentially $\overline{A}$ is a set of everything not in A.

Example

Let $A = \{x \in \mathbb{N} : x < 10\}$. We can infer from the definition of A that $U = \mathbb{N}$.
Then $\overline{A} = \{x \in \mathbb{N} : x \geq 10\}$.

Venn Diagrams

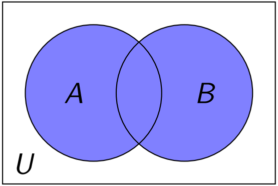
A **Venn Diagram** can help understand set operations on a small number of sets. Closed regions are used to represent sets. Overlapping regions of two or more sets denote elements common to the sets.



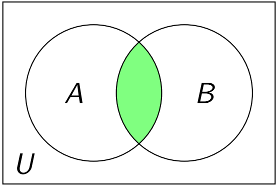
Often the rectangle representing the universal set is not drawn.

Venn Diagrams

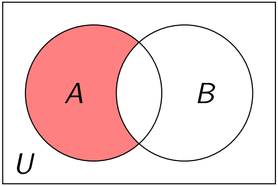
Here are some Venn diagrams of common two-set operations.



$A \cup B$



$A \cap B$



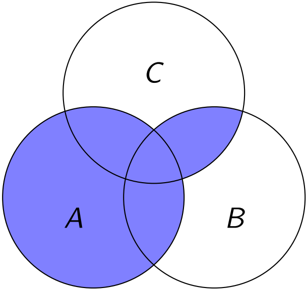
$A - B$

Three Set Venn Diagrams

Construct a Venn diagram for $A \cup (B \cap C)$ and for $A - (B \cup C)$

Three Set Venn Diagrams

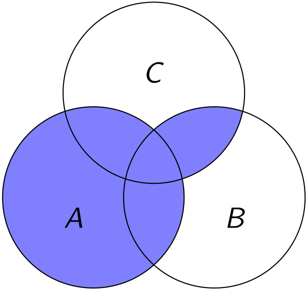
Construct a Venn diagram for $A \cup (B \cap C)$ and for $A - (B \cup C)$



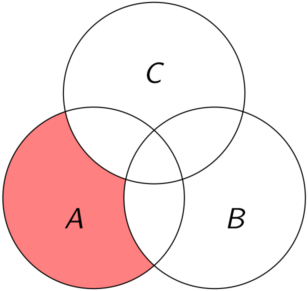
$A \cup (B \cap C)$

Three Set Venn Diagrams

Construct a Venn diagram for $A \cup (B \cap C)$ and for $A - (B \cup C)$



$A \cup (B \cap C)$



$A - (B \cup C)$

Indexed Sets

When working with many sets, it is often convenient to use **indexed sets**. Thus, rather than sets

$$A, B, C, D, E,$$

we might use sets

$$A_1, A_2, A_3, A_4, A_5.$$

We can express the union and intersections of all of these sets as

$$\bigcup_{i=1}^{5} A_i \qquad \text{and} \qquad \bigcap_{i=1}^{5} A_i$$

Indexed Sets

Sometimes an even more general notation is used. Suppose I is any set (including an infinite set). If we use the elements of I as indices then we can have indexed sets A_α where $\alpha \in I$.

Example

$$\bigcup_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for at least one set } A_\alpha \text{ with } \alpha \in I\}$$

$$\bigcap_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for every set } A_\alpha \text{ with } \alpha \in I\}$$