Connecting Voting Theory and Graph Theory

Karl-Dieter Crisman

Gordon College

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Outline

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(Change a few names to get the Minnesota election for governor in 1998, where radio host, wrestler, and small-town mayor Jesse Ventura won.)

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(You can think of lots of other paradoxes, such as the 2000 US presidential election; that example is closely related to Simpson's paradox.)

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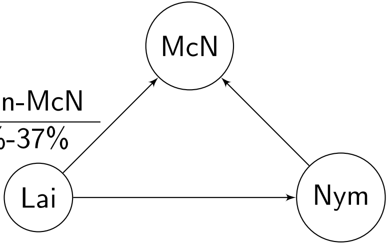

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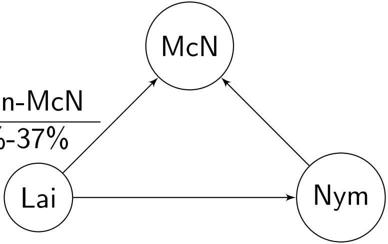
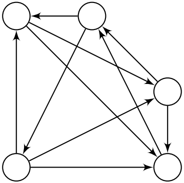


A more general version of this is a 'tournament graph', which can be analyzed for various paradoxes or results.



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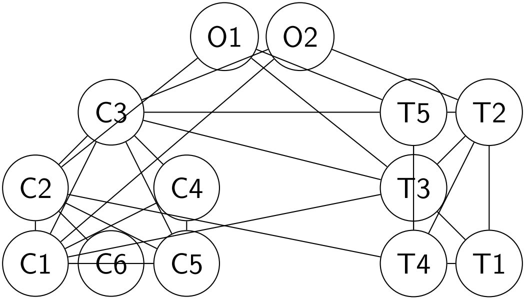

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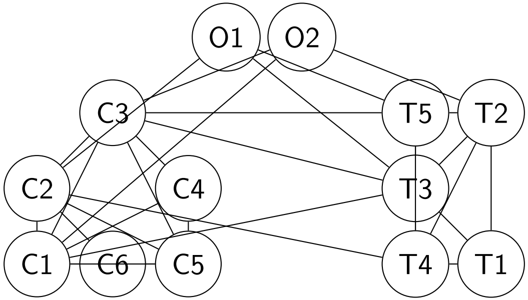

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Bayesian statistical analysis of the polls using graphs may even help Nate Silver predict its outcome ... but I digress!

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- First, relationships among candidates are important:

  In the name of God [the elector should ponder] . . . who among all candidates is least qualified. . . who is next least suitable, and . . . continues until he arrives at the best.

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- Although he is writing far too early to articulate it mathematically, he also cares about the fairness of the method:

  > *It would not be possible to devise a more righteous, just [method in which] . . . the winner is the one who is judged best by the collective verdict of all.*

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This talk explores choosing *graphs* as our modeling tool.

- The choices we care (most) about can be modeled with vertices.
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This talk will introduce a few examples of this productive approach. But don't worry if you don't get every detail – just try for the *flavor* of graphs in analyzing voting!

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Social Preference Functions

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In social choice, there are many frameworks.

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If we ask for both input *and* output to be a full ranking, we call a procedure a *social preference functions*. Potential scenarios include:

- Electing a *full* slate of officers, with succession, for an organization.
- Setting up a rotating schedule for site visits for inspection.

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- The *Borda Count* (BC):
  - With the standard point spectrum of 0 for last up to $n-1$ for first, we order candidates in rank of total points, perhaps with ties.
  - As a social preference function, the outcome is all strict orders compatible with the standard weak order given by the point ranking.
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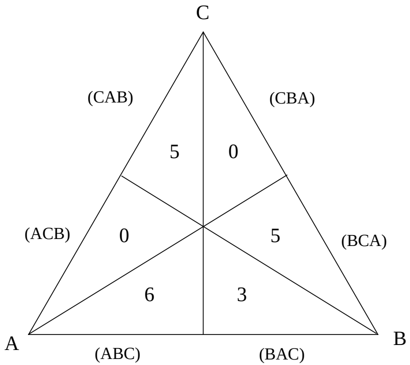
  - (This was Cusa's system! Think college football polls.)
- The *Kemeny Rule* (KR):
  - Look at 'pairwise' votes, like A versus B.
  - For each possible outcome, check for how many pairwise votes it disagrees with each voter.
  - The ranking(s) with the least cumulative disagreement is (are) chosen.

Borda and Kemeny



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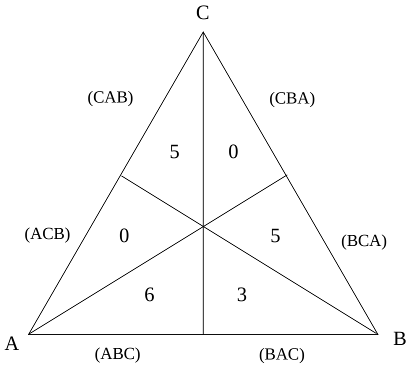

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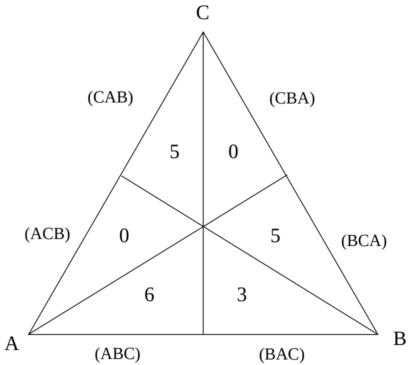

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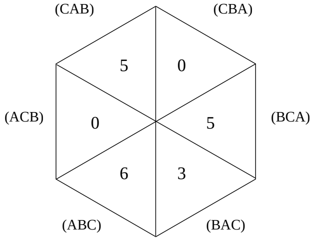
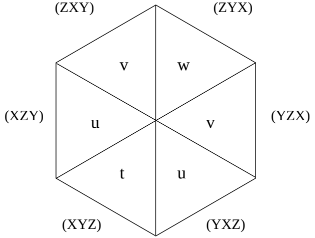
- The Borda count gives the outcome $B \succ A \succ C$, with point totals $22 \succ 20 \succ 15$.
- We'll just highlight a few Kemeny computations:
  - Note that $A \succ C \succ B$ differs in only one pairwise vote from 11 voters, but in all three with 5 voters, totaling 32 points.
  - But $A \succ B \succ C$ agrees completely with six voters and disagrees completely with none, in the end totaling 23 points – and this is the eventual winner.

Towards Graphs





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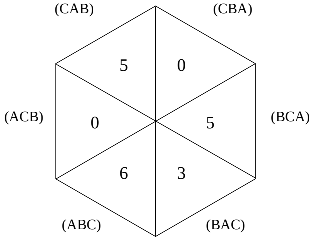

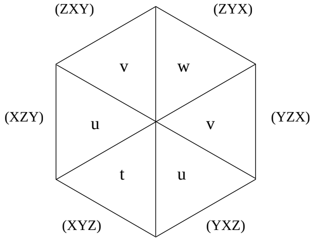

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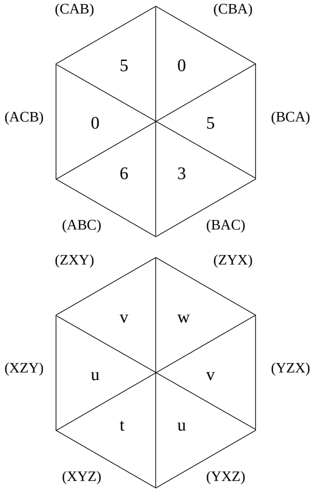

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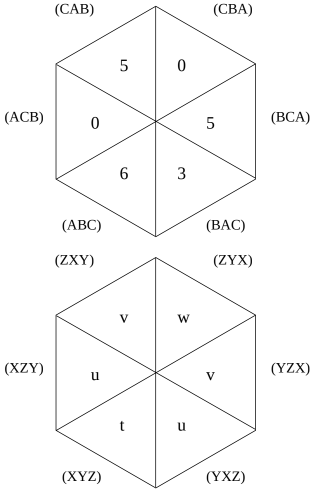

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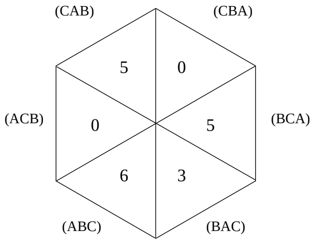

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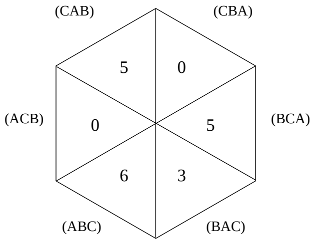

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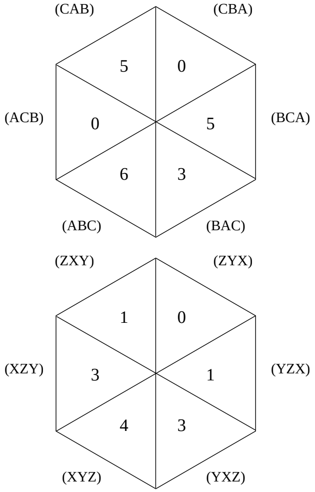

Borda Count Setup

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For instance, this set of weights gives the Borda Count, although it's not obvious. As an example, $B \succ A \succ C$ receives $4 \cdot 6 + 3 \cdot 0 + 3 \cdot 3 + 1 \cdot 5 + 1 \cdot 5 + 0 \cdot 0 = 43$ points; you may wish to take a moment to verify this is the highest possible score.

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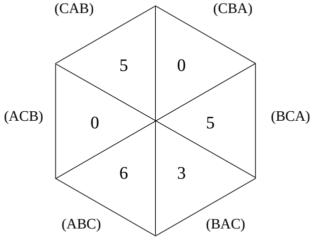

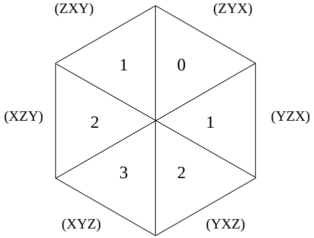

Kemeny Rule Setup

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It's not hard at all to verify that this set of weights gives the Kemeny Rule.

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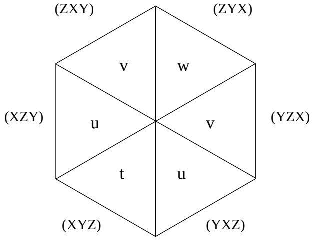

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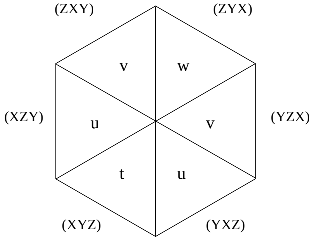

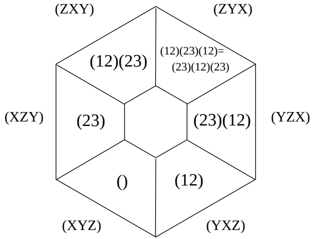

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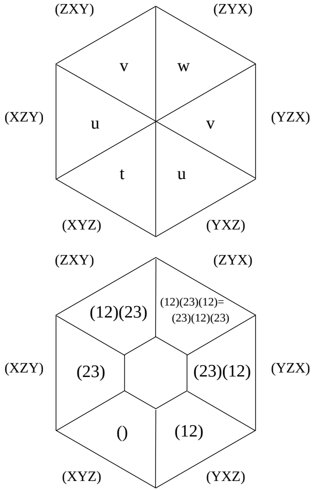

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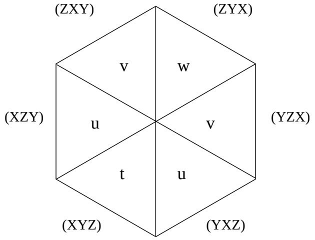

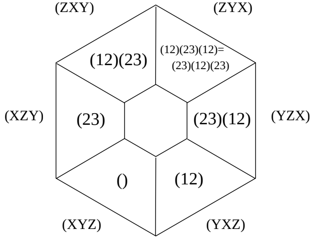

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(You can also think of it as requiring that reversing all preferences would lead to a reversal in the outcome.)

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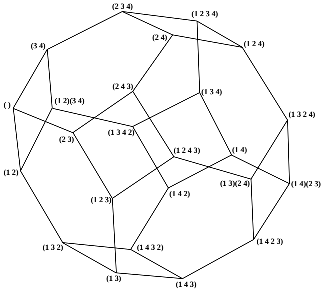
- Borda and Kemeny manage to stick around ...

Graphs and Symmetry



The punch line is that, for any n, we can bring the voting in.

- Create the permutahedron graph, find its symmetries ($S_n \times \mathbb{Z}/2\mathbb{Z}$), and only look at stuff obeying the symmetry.
- Only allow procedures that take head-to-head (pairwise) information into account, if you want.
- **Theorem**: If you then only allow procedures that don't ignore this nice information you just gave it, you now get a procedure 'between' the Borda Count and Kemeny Rule, in a linear algebra sense.

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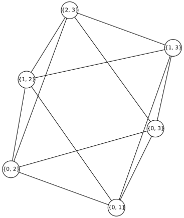
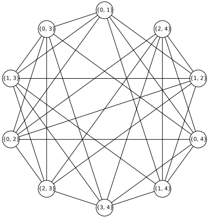
The *Johnson graph* is the graph which proves useful for this model.

The Johnson Graph





The Johnson graph $J(n, j)$ has:

- Vertices which are cardinality-j subsets of $\{1, 2, \ldots, n\}$
- Two vertices joined if the subsets differ in only one element
- Evident connection to votes for a committee in our model

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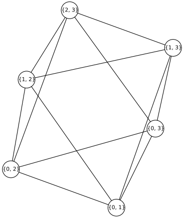

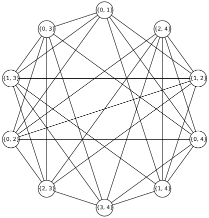

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Note that the graph distance between two vertices v and w is simply the number of candidates differing between the two 'votes'.

Voting on the Johnson Graph

Fact (Davis, Orrison, Su):

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In the examples above, if $j = k = 2$, then for $n = 4$ or 5, unsurprisingly, these are $1/6$ and $1/10$. More interesting is that for $k = 3$ and $n = 5$ we get a guarantee that 30% of the voters will approve of some committee.

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In order to do so, we introduce a notion of a 'ball' around a given vote.

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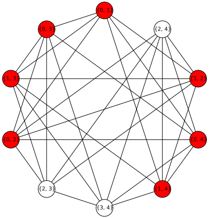
It's easier to see than describe; here is the ball of radius one around $\{0, 1\}$.



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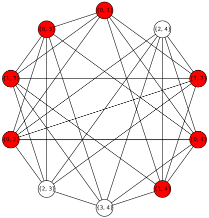

Restricting votes to only be in the red vertices (again, selecting committees of size $k = 3$), this improves the guarantee to $1/3$. In general:

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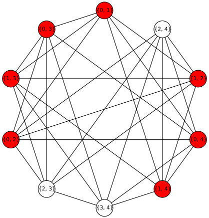

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Theorem (ibid.):

If $\rho \leq j\left[1 - \frac{j}{k+1}\right]$, then if all votes are in a ball of radius ρ, there is a committee approved of by at least $\frac{\binom{k-j}{\rho}}{\binom{n-j}{\rho}}$ of the voters.

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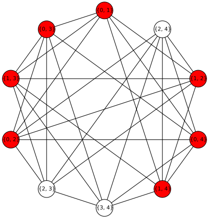

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For instance, if we use the same setup, but with $n = 10$, the first bound gives us $\frac{1}{15}$ while the second gives us $\frac{1}{8}$, which is almost twice as good.

Outline

More Voting Questions

Could we attack other questions where we do not have output a single candidate or choice function? Think again of the example of site visits for inspection.

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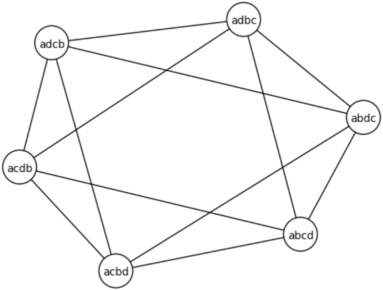
Over the long term, perhaps the cyclic order is most important. The so-called *cyclic order graph* catalogues these. Here is $CO(4)$.



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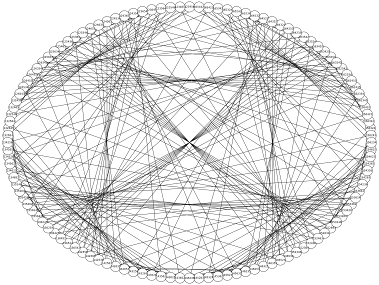
Let's agree not to construct $CO(6)$ by hand.



Symmetry of Cyclic Orders

What *voting* information can we get from these?

- We assume, just like in the committee and ranking case, that certain minimal changes give an edge. Here, it is one swap in the cycle.
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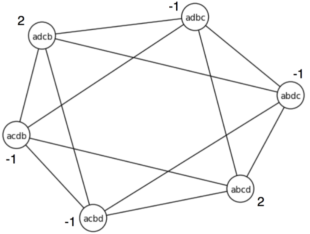
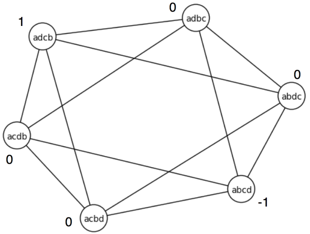
In $CO(4)$, here are the interesting decompositions, which have definite voting flavor.





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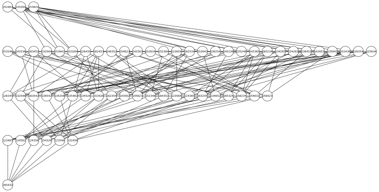
For $n > 5$, it turns out this has half as much symmetry; can we get meaningful information about voting on how to sit at the table from it?



Takeaways and Thanks

What should you remember a week from now about this talk?

- Voting theory can use most interesting mathematics!
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Questions?