Math 112

First Midterm Exam

Instructions: This exam has a total of 100 points. You have 50 minutes. Please show every step, no matter how excruciating, and give the reason why it works - axiom or theorem or whatever. Please use bluebooks, and start a new page for each problem.

(16) 1. A few short answers to begin your test.

   (a) Suppose we have used axioms to show that $(a + bc) - (cb) = a$. Which axioms (and definitions) did we use, in what order?

   (b) List the numbers in one's digit arithmetic which do *not* have a multiplicative inverse. Then write the additive inverse for each of the numbers in your list.

(15) 2. Now, just give the answers - no reasons needed.

   (a) State the ordering axiom O2 (often known as transitivity).

   (b) True or False: The usual arithmetic of $\mathbb{Z}$ satisfies the Cancellation and Well-Ordering Axioms.

   (c) True or False: If $a < b$, then $a^2 < b^2$.

(30) 3. Choose two of the following four items to prove, one from each section. You must prove one labeled i., and one labeled ii.

   (a) Prove one of the following statements, proving from the axioms.

      i. Suppose we have an order (with O1-O4) on a set with addition (A1-A4). Prove that if $a < b$ and $c < d$, then $a + c < b + d$.

      ii. Suppose we have an Abelian group under addition. Prove that if 0 and $0'$ both are additive identities, then in fact $0 = 0'$.

   (b) Choose one of the following items to prove, from axioms and (other) theorems.

      i. In the integers, if $a < b$ and $c < 0$, then $ac > bc$.

      ii. If a is in a commutative ring, then $(-1)a = -a$.

(24) 4. Now some more short answer questions. Give reasons!

(a) Give a solution to the triangle game with one side $\{1, 6, 2\}$. Then give two more solutions which you obtain via one of the transformations we discussed in class or in the book (and say which ones you used). One of these new solutions must have the same side sum, the other must have a different side sum.

(b) Let $A = \{2, \pi, 4, 7\}$ and $B = \mathbb{Z}$. Let $f : A \to B$ be defined by the set

$$\{(4, -2), (7, -3), (2, -1), (\pi, -1)\}.$$

This is a function. Give the domain, range, and image of f, and say whether it is one-to-one and/or onto.

(c) Let your age be denoted by n, and the set of your immediate family be denoted by F. If $A = \{x \in F|\text{ age of } x \geq n\}$ and $B = \{x \in F|\text{ age of } x \leq n\}$, write down explicitly $A \cap B$ and $A \cup B$. Hint: be careful if you have a twin!

(15) 5. Give the answers only.

(a) Give a subset of $\mathbb{R}$ that shows it does *not* have the Well-Ordering property.

(b) Is the set of even integers closed under multiplication? Does it have a multiplicative identity or inverses?

(c) Consider the set $S = \{0, 1, 2, \ldots, 10, 11\}$ arranged as on an analog clock, with 0 replacing 12. Let us define an addition $+$ on S. Start at a o'clock, and move forward b hours; this gives us $a + b$. For instance, $3 + 11$ is eleven hours after 3, which is 2 o'clock. Remember that 12 o'clock is replaced by 0 o'clock! Given all this, what is the additive inverse of 5 under this operation?