Math 112

Second Midterm Exam

Instructions: This exam has a total of 100 points. You have 50 minutes. Please show your work, including the appropriate axiom or theorem if necessary. Please use bluebooks, and start a new page for each problem. The last part of the third question is a bonus, so save that for last.

(15) 1. To begin the test, just answer the question - no reasons needed.

(a) If $n = 31^2$, what is $\sigma(n)$?

(b) List all primes p such that $p \leq 31$.

(c) True or False: the ISBN 0-110-20021-6 is valid.

(d) Is the set $\{0, 1, 2, 3, \ldots, 29, 30\}$ a group under $\pmod{31}$ multiplication?

(30) 2. Here are some short answers - just be sure to show your work or explain your answers.

(a) Describe the six elements of the non-commutative group we looked at in class. You do not need to show it is not commutative! Just describe the group in your own words or pictures.

(b) What is the definition of a composite number? Give an example and tell why it's composite.

(c) How is the Well-Ordering Principle used in the proof of the Euclidean algorithm? If you know what set it is applied to, that would be great to say.

(d) Is the group $\mathbb{Z}_{11}$ cyclic? How about the group $U(11)$? If so, be sure to give generators!

(15+5) 3. Time for work with various algorithms.

(a) Use the Division Algorithm on the numbers $a = 306$ and $b = 12$. (Hint: Long Division)

(b) Now use the Euclidean Algorithm to find the GCD $(306, 12)$.

(c) Now use the Fundamental Theorem of Arithmetic to find the GCD.

(d) (Bonus) What is the order of the answer, as an element in $\pmod{12}$ arithmetic?

(20) 4. Prove one of the following items. You may use other theorems to prove these items.

(a) If $m \geq 2$ and $a \equiv b \pmod{m}$, show that for any $c \in \mathbb{Z}$ we have $ac \equiv bc \pmod{m}$.

(b) Take three integers a, b, c. If $a|b$ and $a|c$, then $a|(b + c)$.

(20) 5. The last section of the test! Show your work.

(a) In the part of the previous problem which you did not prove, give a numerical example for it (e.g. if you proved the *first* part, then find a, b, and c such that the *second* part works).

(b) What is the order of $\mathbb{Z}_6$? What is the order of the element 4 in that group? What are the elements of $U(6)$? If 4 is in $U(6)$, then give the order of 4 in that group as well.

(c) Find the highest power of 2 dividing the number 12897192, as well as whether it is divisible by 3, 9, or 11. (It is not divisible by 5!)